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AN ACCURATE EXPERIMENTAL INVESTIGATION OF THE SHOCK-TUBE END-WALL BOUNDARY LAYER IN ARGON

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Abstract—A laser schlieren method is applied to measure the density-gradient profile of the shock-tube end-wall boundary layer in argon. From measurements at different positions the expected self-similar structure of the boundary layer is well established. Corrections are made for the effects of higher-order spatial derivatives of the density on the schlieren signal. The experimental boundary-layer structure is found to agree within an error bound of 2% with the theoretical calculation based on recent accurate thermal-conductivity data, if the variation of the pressure is taken into account.

NOMENCLATURE

a, thermal diffusivity;
F, reduced density gradient;
I, radiant flux density;
k_0, wave number of the laser beam;
L, width of the shock tube;
M, shock Mach number;
N, refractive index;
N_i, ith order space derivative of the refractive index;
p, pressure;
p_1, initial testgas pressure;
p_{RH}, reflected shock pressure based on the Rankine—Hugoniot relation;
R, specific gas constant;
s, similarity coordinate: $x/(a_{RH}t)^{1/2}$;
t, time;
T, temperature;
T_c, temperature of the contact plane between gas and wall;
w_0, waist of the laser beam;
x, space coordinate along the shock tube axis;
x_m, distance of beam center to endwall;
y, space coordinate;
z, space coordinate along the undisturbed laser beam axis;
z_0, characteristic diffraction length;
z_D, position of the photodetector.

Greek symbols

η, Lagrangian coordinate;
ϕ, schlieren signal;
ρ, gas density;
λ, thermal conductivity.

Subscripts

∞ state of the gas outside the boundary layer;
RH, value based on ideal Rankine–Hugoniot theory;
th, theory;
ex, experiment.

Superscript

(1), first order estimate.

INTRODUCTION

IN RECENT years much progress has been made in the experimental determination of thermal conductivities of gases by means of hot-wire methods. The experimental inaccuracy has been substantially reduced and the temperature range has been extended. Chen and Saxena [1] were able to determine the thermal conductivity of argon within 1.5% for temperatures between 350 K and 2500 K. The use of a shock tube in thermal-conductivity studies seems most appropriate for the high temperature range. Basically, two different approaches can be followed. The heat flux from the shock-heated test gas to the wall causes a small increase in contact temperature, that can accurately be measured and from which thermal-conductivity data can be derived. Saxena [2], reviewing the determination of thermal conductivity data in this manner, concludes that the experimental values have an inaccuracy of the order of 10–15% and that the method is subject to the restriction that independent thermal-conductivity data at lower temperatures are needed.

No such restriction has to be made, when the structure of the thermal boundary layer in the test gas is determined and the thermal conductivity is obtained by fitting theoretical and experimental profiles. So far, the results obtained in this way are not very satisfactory, since significant differences are found with the hot-wire values for the thermal conductivity in the overlapping temperature range [3].

In the present investigation, a laser-schlieren method is applied to determine the thermal boundary-layer structure in argon for conditions under which the transport properties are accurately known. In such a case, a detailed comparison between theory and experiment can be made.

THEORETICAL BOUNDARY LAYER STRUCTURE

A discussion of the boundary-layer structure, including possible effects of thermal accommodation and the interaction between boundary layer and the reflected shock wave is given by Clarke [4]. We shall assume that the boundary layer is developing in a uniform semi-infinite reflected shock region, indicated by means of a subscript '∞'. The energy equation for a perfect gas can be written in Lagrangian coordinates (η, t) as:

$$\frac{\partial T}{\partial t} = a_\infty \frac{\partial}{\partial \eta}\left(\frac{\lambda}{\lambda_\infty} \frac{T_\infty}{T} \frac{\partial T}{\partial \eta}\right) \tag{1}$$

with:

$$\eta = \frac{1}{\rho_\infty} \int_0^x \rho(x')\mathrm{d}x' \tag{2a}$$

or

$$x = \rho_\infty \int_0^\eta \frac{1}{\rho(\eta')} \mathrm{d}\eta', \tag{2b}$$

where x is the distance from the point considered to the endwall (see Fig. 1) and t is the time. Use has been made of the ideal gas law, $p = \rho RT$, and of the symbols T for temperature, ρ for density, λ for thermal conductivity, a for thermal diffusivity, and p for pressure. The initial and boundary conditions are:

$$\begin{aligned} t &= 0, \quad \eta > 0: \quad & T &= T_\infty, \\ t &> 0, \quad \eta = 0: \quad & T &= T_c, \\ t &> 0, \quad \eta \to \infty: \quad & T &\to T_\infty. \end{aligned} \tag{3}$$

It can be shown that the contact temperature T_c of the gas at the wall is constant and slightly higher than the ambient temperature [1]. The temperature T appears to be a function of the similarity variable $\eta/t^{1/2}$ only.

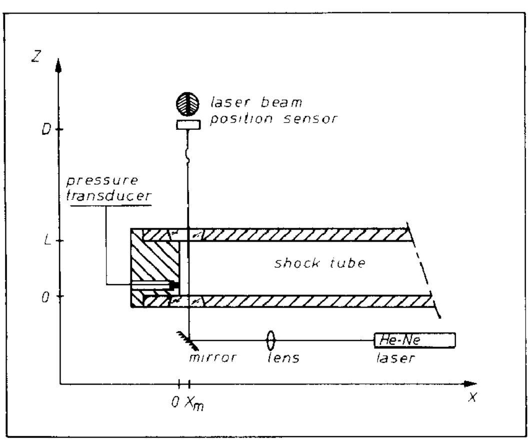

FIG. 1. Schematic view of shock tube and laser-schlieren system.

The partial differential equation (1) then reduces to an ordinary differential equation, that can easily be solved for any specific $\lambda(T)$ relation.

But, if temperature and density are functions of $\eta/t^{1/2}$, it follows directly from equation (2b), that temperature and density are functions of $x/t^{1/2}$. In view of this we now introduce functions s and F as follows:

$$s = x/(a_{RH}t)^{1/2} \tag{4}$$

and

$$F(s) = \frac{-1}{\rho_{RH}}(a_{RH}t)^{1/2}\frac{\partial \rho}{\partial x}. \tag{5}$$

The shape of $F(s)$, the boundary-layer structure, depends on the $\lambda(T)$ relation. Following Chen and Saxena [1], we use the following empirical result:

$$\begin{aligned} \lambda(T) = (5.465 + 4.729 \times 10^{-2}T - 1.111 \\ \times 10^{-5}T^2 + 1.599 \times 10^{-9}T^3) \times 10^{-3}\mathrm{Wm^{-1}K^{-1}} \\ [T] = \mathrm{K}, \end{aligned} \tag{6}$$

which should approximate the thermal conductivity of argon within 1.5% for temperatures between 350 K and 2500 K.

EXPERIMENTAL METHOD

The boundary-layer structure has been investigated experimentally by means of the schlieren deflection of a laser beam (Fig. 1). Since the schlieren deviation and the beam width are not much smaller than the boundary-layer thickness, the effects of higher order space derivatives of the refractivity should be taken into account. To analyse the propagation of the laser beam through the boundary layer, a second order perturbation theory has been applied based on the free-space modes of Kogelnik [5].

The undisturbed laser beam is adjusted parallel to the end wall and focussed at the center of the test section ($z = L/2$) in order to reduce the lens-effect of the inhomogeneity on the beam. Its radiant flux distribution in the focal plane is:

$$I(x, y, L/2) = I_0 \exp\left\{-\frac{2[(x - x_m)^2 + y^2]}{w_0^2}\right\}. \tag{7}$$

The parameter w_0 is the so-called waist of the beam and x_m denotes the distance of the beam center line to the end-wall. The deflection of the beam is measured by means of a photodetector, consisting of two semi-circular segments, placed at position D. This set-up is described by Diebold [6]. The signal obtained is proportional to ϕ defined as:

$$\phi = \int_{-\infty}^{\infty}\int_{x_m}^{\infty} I(x, y, D)\mathrm{d}x\,\mathrm{d}y - \int_{-\infty}^{\infty}\int_{-\infty}^{x_m} I(x, y, D)\mathrm{d}x\,\mathrm{d}y. \tag{8}$$

Representing the boundary-layer profile by means of a Taylor series of the refractive index N around x_m, and denoting the ith order derivative by N_i, we obtained for ϕ:

$$\lim_{D\gg L, z_0} \phi = \frac{k_0^2 w_0 I_0 L}{(2\pi)^{1/2}}\Bigg[\underbrace{N_1\left(1+\frac{N_2L^2}{6}\right)}_{A} + \frac{N_3 w_0^2}{6}\left(1+\frac{N_2L^2}{16}\left(7+\frac{3}{20}\left(\frac{L}{z_0}\right)^2 + \frac{1}{336}\left(\frac{L}{z_0}\right)^4\right)\right)+\cdots\Bigg] \quad (9)$$

The characteristic diffraction length z_0 is related to the waist w_0 and the wave number k_0: $z_0 = \frac{1}{2}k_0 w_0^2$. Term A of equation (9) can be directly obtained from geometrical-optics considerations. The remaining terms are typical results of the paraxial wave analysis of the problem.

SHOCK TUBE EXPERIMENTS

A series of shock-tube runs in argon has been performed for a restricted Mach number range (2.715–2.762) corresponding to a reflected shock-temperature range of 1775–1825 K for an initial temperature of 295 K. The initial pressure was 667 ± 3 Pa, measured by means of an oil filled vacuum micro-manometer. The experiments were done in a $10 \times 10\,\text{cm}^2$ steel shock tube. Leak- and outgassing rate were such that the impurity level was estimated to be less than 2.10^{-4}. A helium–neon laser was used as a light source. The waist w_0 was chosen according to: $w_0 = (1/3)^{1/4}(L/k_0)^{1/2}$, which corresponds to a minimum average beam width in the test section. The position of the beam x_m has been varied from 0.4 to 2 mm with an inaccuracy less than 0.01 mm. The displacement of the shock tube during the experiment was measured and taken into account.

For the data reduction the following iterative procedure was adopted. From the experimental values of ϕ, the corresponding values of the gradient N were found by using the approximate formula $\phi = k_0^2 w_0 I_0 L N_1/(2\pi)^{1/2}$. In this way the function $F^{(1)}$ (s) of Fig. 2a was obtained, using for the specific refractivity of argon the value: $1.582.10^{-4}\,\text{m}^3\,\text{kg}^{-1}$. For reason of survey only four out of nine experiments are shown, which are representative for the whole series. Then a best-fit polynomial was determined from six experimental runs for which x_m exceeded 0.59 mm. The exception was made since for the lowest values of x_m (0.495, 0.445 and 0.395 mm), even fifth order derivatives of the refractive index contribute to the signal. From this polynomial first estimates were derived from N_2 and N_3. Then, with equation (9) corrected values for N_1 were found. After two iterations the procedure resulted in the corrected values for $F(s)$ of Fig. 2b, which shows a substantial reduction of the scatter compared with Fig. 2a.

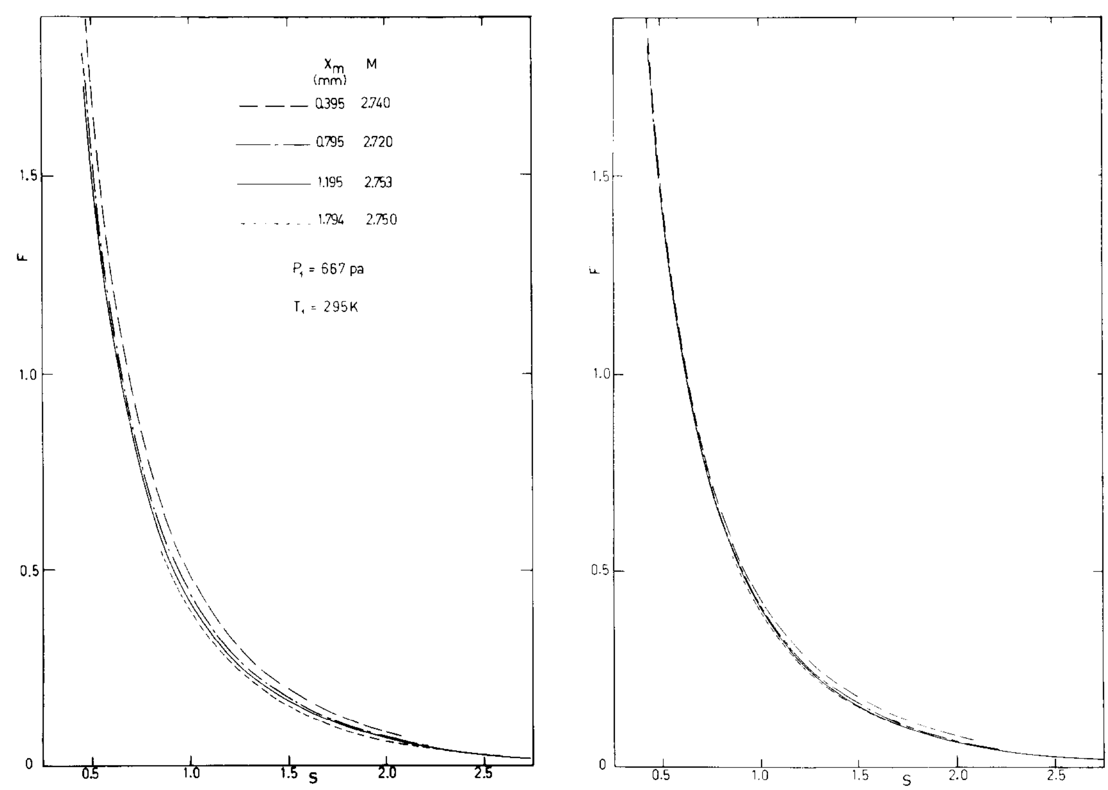

FIG. 2. Experimental results; reduced density gradient as a function of similarity coordinate for various measuring positions; (a) only first order schlieren effects are taken into account; (b) data are corrected for second and third order derivatives of the refractive index.

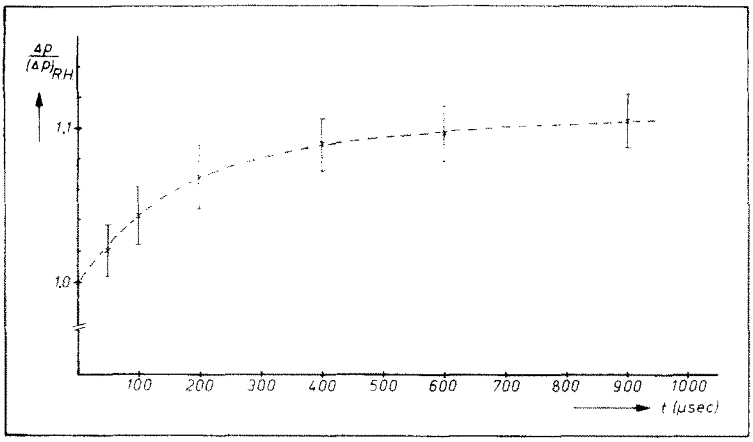

FIG. 3. Relative deviation of pressure from Rankine–Hugoniot value as a function of time.

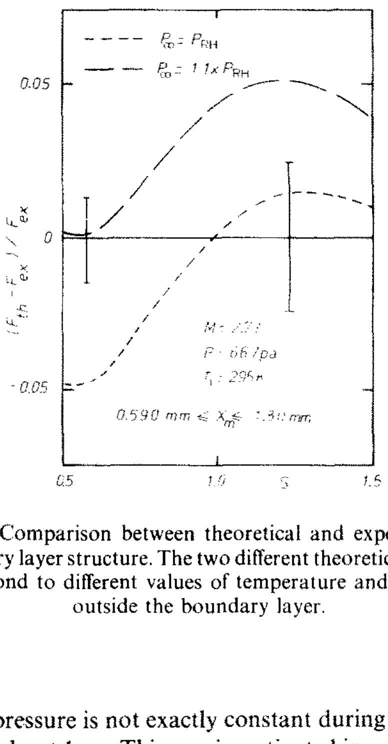

FIG. 4. Comparison between theoretical and experimental boundary layer structure. The two different theoretical curves correspond to different values of temperature and pressure outside the boundary layer.

The pressure is not exactly constant during the test time of about 1 ms. This was investigated in a series of eleven shock-tube runs by means of a piezo-electric pressure transducer. The transducer sensitivity was determined by a dynamic calibration procedure. The relative deviation of the pressure from its Rankine–Hugoniot value is depicted in Fig. 3 as a function of time. Pressure appears to increase from its ideal Rankine–Hugoniot value to a value that is about 10% higher after 1 ms. For that reason theoretical calculations were determined for two different conditions: the Rankine–Hugoniot pressure and temperature, and for a 10% higher pressure with a corresponding isentropically-increased temperature. The theoretical boundary-layer profiles were evaluated for the average Mach number of 2.73 and the initial pressure and temperature of 667 Pa and 295 K, respectively. For the experimental boundary layer profile, the best fit polynomial after two iterations is:

$$F_{ex} = -0.04791 + 0.4978\,s^{-2} - 0.02487\,s^{-4}. \quad (10)$$

The relative deviation between experiment and theory and the corresponding error bars are shown in Fig. 4.

For large values of s (≈ 1.5), that is for short times, the experiment agrees within its error bounds of about 2% with the theoretical curve based on the ideal Rankine–Hugoniot pressure. For small values of s (≈ 0.5), the experiment agrees with the solution corrected for the pressure increase, within the error bounds of about 1%.

CONCLUSIONS

By applying a laser schlieren method it is possible to determine the structure of the shock-tube thermal boundary layer within an error bound of 1–2%. It is necessary then to make corrections for the effects of higher order space derivatives of the refractive index on the schlieren signal.

Besides, the state of a shock-heated gas can usually not be considered as constant.

The changing pressure and temperature outside the boundary layer have a significant effect on the boundary-layer structure. If this effect is taken into account, a good agreement is found between the present experiments and a boundary-layer theory based on the thermal-conductivity data of Chen and Saxena.

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Résumé—Une méthode strioscopique à rayon laser est utilisée pour la mesure du gradient de la densité dans la couche thermique pariétale dans de l'argon, à l'extrémité d'un tube à choc. Une série de mesures effectuées à différentes positions apporte une vérification expérimentale de l'auto-similarité de la structure de la couche limite, prévue par la théorie. L'estimation du gradient de la densité à partir de la mesure de la déflection du rayon laser est basée sur une théorie tenant compte des dérivées spatiales de l'indice de refraction d'ordre supérieur. La structure de la couche limite déterminée expérimentalement est en bon accord avec les résultats numériques théoriques obtenus sur la base de données récentes sur la conductivité thermique de l'argon, si la variation de pression est prise en considération.

EINE GENAUE EXPERIMENTELLE UNTERSUCHUNG DER GRENZSCHICHT AN DER STIRNSEITE EINES STOSSWELLENENDROHRES MIT ARGONFÜLLUNG

Zusammenfassung—Eine Laserschlierenoptik-Methode wurde verwendet, um den Verlauf des Dichtegradienten der Grenzschicht an der Stirnseite eines Stoßwellenendrohres beim Betrieb mit Argon zu messen. Messungen an verschiedenen Stellen bestätigten gut die erwartete Ähnlichkeitsstruktur der Grenzschicht. Korrekturen des Schlierensignals in Bezug auf die räumlichen Differentialkoeffizienten höherer Ordnung der Dichte wurden vorgenommen. Die experimentell gefundene Grenzschichtstruktur ist bis auf eine Abweichung in Höhe von 2% in Übereinstimmung mit den theoretischen Berechnungen, die sich auf neue genaue Wärmeleitfähigkeitsdaten stützen, wenn die Druckänderung berücksichtigt wird.

ТОЧНОЕ ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ПОГРАНИЧНОГО СЛОЯ АРГОНА В УДАРНОЙ ТРУБЕ

Аннотация — С помощью шлирен-метода измерен профиль градиента плотности пограничного слоя аргона в ударной трубе. Результаты измерений, проведенных в различных точках, позволили получить предполагаемую автомодельную структуру пограничного слоя. Сделаны поправки на влияние на шлирен-сигнал пространственных производных высших порядков по плотности. Найдено, что экспериментальные данные о структуре пограничного слоя согласуются с точностью до 2% с теоретическими расчетами, проведенными на основании полученных недавно точных данных о теплопроводности при учете изменения давления в пограничном слое.